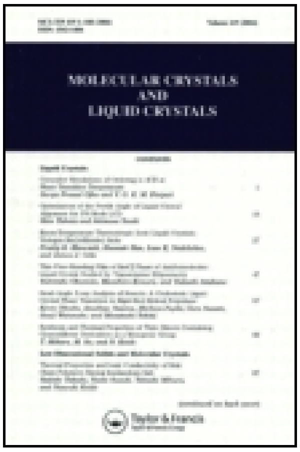

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Effect of Fluctuations on Evanescence in Nematics

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EFFECT OF FLUCTUATIONS ON EVANESCENCE IN NEMATICS

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The effect of orientational fluctuations in nematic liquid crystals on the existence of evanescent modes of a transmitted electromagnetic waves is discussed. It is shown that the ratio of evanescent to all (homogeneous and evanescent) modes is a function of the orientational distribution function. The ratio allows further insight on the determination of the orientational potential energy function.

The existence of evanescent waves in liquid crystals, created by the incidence of an arbitrary electromagnetic field has been recently presented by Moritz[1]. Striking features of evanescent wave propagation include the fact that surfaces of constant phase may be at arbitrary angles to surfaces of constant amplitude, while at the same time the evanescent mechanism precludes net energy transfer. The importance of evanescent wave amplitudes and their nature in the structure of general fluids has been recognized by Hertzfeld[2] .

It has been pointed out in reference 1 that there will be oscillations in the ratio of evanescent to homogeneous modes in direct correspondance with the order parameter. This letter develops the methodology for determining the depen- ce of the transmitted wave on the orientational fluctuations and consequently the orientational distribution function for a simple Saupe-Maier type theory.

Particular interest in the effect of fluctuations on evanescence is generated by the need to understand director fluctuations in lyotropics, for instance DPPC-water mixture, which have two-dimensional phases[3]. The DPPC-water mixture is similar to a two-dimensional nematic where the orientation of the bond direction is the physical analog of the molecular axis in a thermotropic nematic. Thus, results developed for conventional nematics may also have applications in lyotropic systems and, by extension of Brown and Wolken's results and conjectures, to biological strucres.[4]

The remainder of the letter utilizes the conventions and nomenclature of reference 1, where the coefficients $n_i (i=1,2)$ presented in the expression for s^2 are defined by the rule $n_i=m_i/a_i$ and where m_i are integers and a_i are characteristic lengths of the liquid crystal. These may arise from the particular boundary conditions imposed as well as material parameters (the lack of experimental data in this area makes appropriate identification difficult).

In real systems there exist fluctuations in a variety of quantities such as density, temperature, as well as orientational fluctuations. We wish to restrict ourselves to consider those fluctuations in orientation that are described by the orientational distribution function.[5] Specifically the orientational distribution function $\rho(\cos\alpha)$ describes the probability of finding a molecule at a prescribed angle α from the director $\vec{n}$. The functional dependence of ρ is given by:

$$\rho(\cos\alpha) = Z^{-1}\exp\{-\beta V(\cos\alpha)\}$$

where $\beta=1/kT$, k being Boltzman's constant, T the temperature, V the potential function, and Z the partition function given by:

$$Z = {}_0\int^1 d(\cos\alpha)\exp\{-\beta V(\cos\alpha)\}.$$

The conditions for transmitted evanescent waves are[1]:

$$s^2 = \eta_o^{-2}(p^2+q^2)>k^2 \quad \text{for ordinary rays}$$

$$s^2 = \eta_e^{-2}(p^2+q^2)>k^2 \quad \text{for extraordinary rays,}$$

accordingly one may define the threshold values of s as $s_{o,th} = \pm\eta_o k$ and $s_{e,th} = \pm\eta_e k$ as threshold values such that when s exceeds these values the appropriate transmitted wave is evanescent.

If we now assume a particular choice of n_i, determined experimentally, such that for a given angle θ_0 between the incident electromagnetic field and the director, the condition for evanescent (ordinary) transmitted wave is:

$$s^2_{o,th} = (\lambda_1 n_1 + \sin\theta_0 \cos\phi_k)^2 + (\lambda n_2)^2$$

then it is obvious that when $\sin\theta > \sin\theta_0$ all waves will be evanescent while angles where this is not true will lead to homogeneous modes.

Let $0 \leq \theta_0 < \pi/2$ and the actual fluctuating angle between the incident field and the director be $\theta = \theta_0 + \alpha$, we are now interested in finding the probability of finding α such that $0 \leq \alpha \leq \pi/2 - \theta_0$. It is now easy to see that R_e defined by:

$$R_e = Z^{-1} \times \int_0^{\cos\gamma} d(\cos\alpha)\rho(\cos\alpha) \qquad (1)$$

is the ratio of evanescent to all possible modes when $\gamma = \frac{1}{2}\pi - \theta_0$.

Figure 1 depicts the results of calculation for R_e vs. γ using the simple Saupe-Maier theory where the potential function V has the form $V = -A_0 S(3\cos^2\alpha - 1)/2$ where A_0 is a quantity which depends on interparticle spacing but not on orientation and S is the order parameter (uniaxial liquid crystals). The results are presented for the cases where A_0S/kT = 0.2, 2.0, and 20.0. The case of $A_0S/kT = 2.0$ is essentially the value predicted by the Maier-Saupe theory for a first-order phase transition[6].

The results of parametric variations of S,T of an extensive nature are left to future communications, however it is evident that in the neighbourhood of a phase transition the ratio R_e increases exponentially, for all cases an incident wave parallel to the director n will always produce evanescent waves for the appropriate boundary conditions(these are then simple electromagnetic surface waves) while large values of S/T lead to vanishing contributions from evanescent modes for all angles not close to grazing angles with the director.

It is also possible from this curve to note that the derivative $\partial R_e/\partial(\cos\gamma) = \rho(\cos\gamma)$, thus presenting an

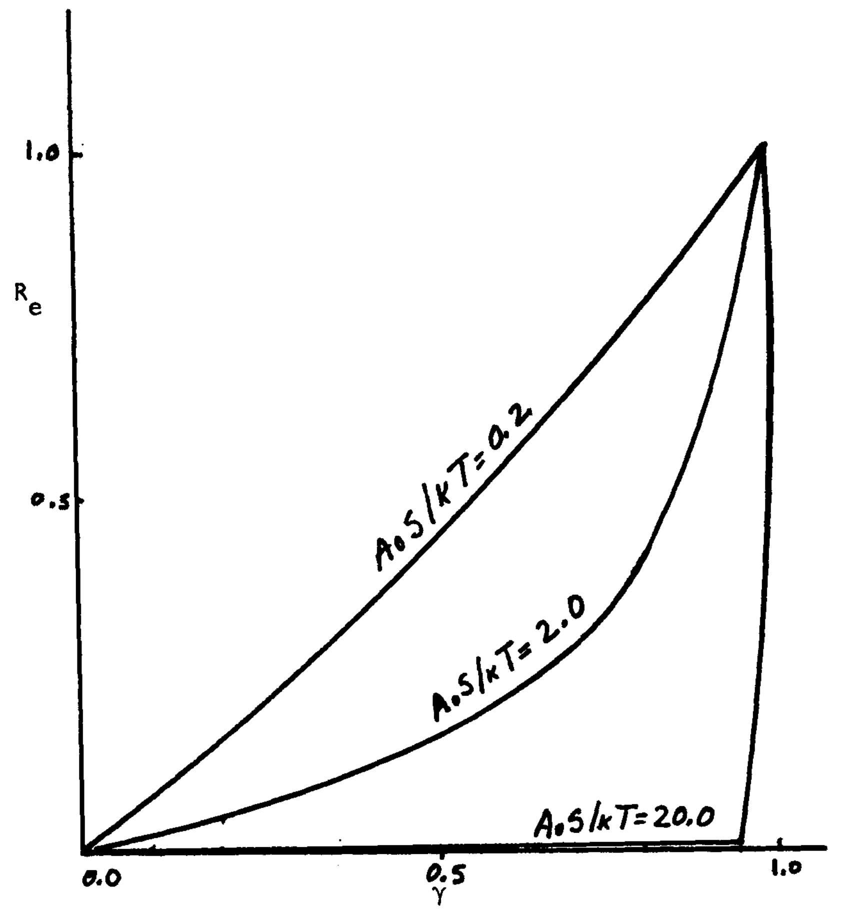

FIGURE 1 The dependence of R_e (ratio of evanescent to all modes) on γ.

additional method for obtaining the orientation distribution function.

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